GCE

Further Mathematics A

Y533/01: Mechanics

Advanced Subsidiary GCE

Mark Scheme for June 2019

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question includes the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for AS Level Further Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to **3 s.f.** unless the question specifically asks for another level of accuracy.
- Follow through should be used so that only one mark is lost for each distinct accuracy error.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1	(a)		Initial (kinetic) energy = $\frac{1}{2} \times m \times 8.4^2$	B1	1.1a	$35.28m$	m may be implied
			Energy at 0.8 rad = $\frac{1}{2}mv^2 + m \times 9.8 \times 2.5(1 - \cos 0.8)$ = Initial energy	M1	1.1	Attempt to find KE + PE at 0.8 rad (or 45.8°) and equate to initial kinetic energy (KE must use correct formula) NB $\Delta h = 0.7582$ ...	$\frac{1}{2}mv^2 + 7.4306...m$ Or subtract PE from initial KE (to give final KE) (Final KE is $27.849\ldots m$)
			$v^2 = 55.698... \Rightarrow$ speed is 7.46 m s^{-1}	A1 [3]	1.1	SC1 for use of constant acceleration without justification	
1	(b)		Minimum energy to reach top = $m \times 9.8 \times (2 \times 2.5)$	M1	1.1a	Or attempt to find angle when $v = 0$ $35.28m = 24.5m(1 - \cos\theta)$ $(+½m(0)^2)$ Condone missing m	Or attempt to find h when $v = 0$ $\left(h = \frac{35.28}{g}\right)$
			$= 49m$	A1	1.1	θ = 2.03 rads or 116°	$h = 3.6$
			$49m > 35.28m$ so insufficient energy to reach top	A1ft [3]	2.2a	Comparison between their numerical multiples of m (m could be missing) Allow ≠ and consistent ft conclusion	or comparison of their angle with 2π or 180° Or show that $h = 3.6 < 5$ or show that $v^2 = -27.44 < 0$ (is not valid)

2	(a)		$3.6 \times 7.2 = 3.6v_A + 2.4v_B$ $v_A = v_B$ $4.32\,\text{m s}^{-1}$	M1 M1 A1 **[3]**	1.1a 1.1 1.1	Conservation of momentum soi May be -4.32 if the initial velocity is counted as negative.	(25.92)
2	(b)		$\pm 3.6 \times 4.32 \mp 3.6 \times 7.2$ $-10.4\,\text{N s}$ (or kg m s^{-1})	M1 A1 **[2]**	1.1a 1.1	Using their 4.32 from 2(a) provided c.o.m. used Or 10.4 N s towards *B* Must be opposite sign to the initial velocity.	Or –(2.4 × 4.32) Deduct final mark if correct direction not soi
2	(c)		$\pm\left(\frac{1}{2}\times 3.6\times 7.2^2 - \frac{1}{2}\times(3.6+2.4)\times 4.32^2\right)$ 37.3 J	M1 A1 **[2]**	1.1a 1.1	Using their 4.32 from 2(a) provided c.o.m. used Allow one slip in substitution other than sign error; must have 3 terms Allow -37.3J	93.31 … – (33.59 … + 22.39 …)

3	(a)		$\frac{60\,000}{10} - R = 1500 \times 3.3$	M1	3.3	= 4950	
			$R = 1050$	A1	1.1	May be -1050	
			$\frac{60\,000}{v} = 1050$	M1	3.4		
			The greatest speed is $57.1\ \text{m s}^{-1}$	A1 [4]	1.1		
3	(b)		$\frac{60\,000}{10} - k \times 10 = 1500 \times 3.3$	M1	3.3	Or $1050 = 10k$	
			$k = 105$	A1	1.1		
			$\frac{60\,000}{v} = 105v$	M1	3.4		
			$v^2 = 571.4...$	A1	1.1		
			$v = 23.9\ \text{m s}^{-1}$	A1 [5]	1.1	Must be positive	
3	(c)		The constant resistance model does not seem to be very accurate	B1ft	3.5a	B1 for each of two correct statements about the *models*. If commenting on the accuracy of (a), must emphasise that (a) is very inaccurate or at least quite inaccurate Do not allow e.g. • model (a) is not very effective • Neither model is accurate • (a) and (b) are not very accurate	Suitable comments for (a): • is very inaccurate • predicted speed is nearly three times the actual value • constant resistance is not a suitable model • both models underestimate the resistance (as top speed is lower than expected)
			The refined (linear) model (is not perfect but) gives a much more accurate answer than the constant resistance model	B1ft	2.4	Clear comparison between the accuracy of the two models (must emphasise that (b) is fairly accurate or considerably more accurate than (a)), or other suitable distinct second comment	For the linear model (b) • is fairly accurate (but probably underestimates the resistance at higher speeds)

				[2]		Do not allow e.g. • model (b) is more accurate than model (a) • (b) is not accurate Do not allow statement claiming that resistance is proportional to speed, or to speed^2	• resistance is not proportional to speed but is a much better model than constant resistance

4	**(a)**		$[\rho] = \mathrm{ML^{-3}}$	**B1** **[1]**	**3.3**	If M, L and T not used B0, but do not penalise any further instances of non-standard notation as long as it is used consistently.	
4	**(b)**		$[p] = \mathrm{MLT^{-2}L^{-2}} = \mathrm{ML^{-1}T^{-2}}$	**B1**	**2.1**	If M, L and T not used B0, provided this has not been withheld in part (a). Award remaining marks where working is clear.	
			$\mathrm{LT^{-1}} = \mathrm{M^{\alpha}L^{-\alpha}T^{-2\alpha}M^{\beta}L^{-3\beta}L^{\gamma}}$	**B1ft**	**3.3**	Or $(MLT^{-1}T^{-2})^{\alpha}(ML^{-3})^{\beta}(L)^{\gamma}$ ft their expressions for p and ρ here and in subsequent method marks provided M, L and T present with M and L appearing at least twice on the RHS Ignore use of k in the equation.	Do not allow any marks for using addition instead of multiplication
			M: $\alpha + \beta = 0$	**M1**	**3.4**		
			T: $-2\alpha = -1$	**M1**	**3.4**		
			$\alpha = \frac{1}{2}$, $\beta = -\frac{1}{2}$	**A1**	**1.1**	Allow this mark as long as the equations for M and T are correct.	
			L: $1 = -\alpha - 3\beta + \gamma$	**M1**	**3.4**		
			$\gamma = 0$ www	**A1** **[7]**	**1.1**	SC2 for three correct values unsupported or SC1 for correct values of α and β or for γ. These may not be combined with any other marks.	

4	**(c)**		Sounds of any wavelength have the same speed through the gas	**E1FT** **[1]**	**2.2b**	Follow from their γ: If γ > 0 then speed increases as wavelength increases (or better – e.g. γ = ½ → speed is proportional to √λ); if γ < 0 then speed decreases as wavelength increases (or better)	
4	**(d)**		$u \propto \frac{1}{\sqrt{\rho}}$ or $u = k\sqrt{\frac{p}{\rho}}$ oe $\frac{1}{4}$	**M1** **A1** **[2]**	**3.4** **1.1**	$2 = \left(\frac{\rho_B}{\rho_A}\right)^{-\frac{1}{2}}$ Award if no working seen provided β = -½	Using their value of β Allow missing k or use of = instead of proportion symbol

5	(a)		$T_2 \cos\theta = m_2 g$ $T_2 = \frac{m_2 \times 9.8}{0.8} = 12.25 m_2$	M1 A1 [2]	1.1a 1.1	Resolving T_2 vertically and balancing forces on R Do not allow extra forces present Allow use of g, e.g. $\frac{5}{4} g m_2$	In this solution θ is the angle between RP and RA Sin may be seen instead if θ is measured horizontally. Do not allow incomplete expressions e.g. $\frac{m_2 g}{\sin 53.13}$
5	(b)	(i)	$T_2 \cos\theta + m_1 g = T_1 \cos\theta$ $T_1 = T_2 + \frac{9.8 m_1}{0.8} =$	M1 A1 [2]	3.1b 2.1	Vertical forces on P; 3 terms including resolving of T_1; allow sign error **AG** Dividing by $\cos\theta$ (= 0.8), substituting their T_2 and rearranging Allow 12.25 instead of $\frac{49}{4}$	Or $T_1 \cos\theta = m_1 g + m_2 g$ (equation for the system as a whole) At least one intermediate step must be seen
5	(b)	(ii)	$T_1 \sin\theta + T_2 \sin\theta = m_1 a$ $12.25(m_1 + m_2) \times 0.6 + 12.25 m_2 \times 0.6 = m_1 \times 0.6\omega^2$ $\omega^2 = \frac{7.35 m_1 + 14.7 m_2}{0.6 m_1} = \frac{49(m_1 + 2m_2)}{4 m_1}$	M1 M1 A1 [3]	3.1b 1.1 2.1	NII horizontally for P; 3 terms including resolving of tensions; allow sign error Substituting for T_1, their T_2, $\sin\theta$ and a **AG** Must see an intermediate step	Could see a or $0.6\omega^2$ or $\frac{v^2}{0.6}$ or $\omega^2 r$ or $\frac{v^2}{r}$ $\sin\theta = 0.6$ must be $a = 0.6\omega^2$
5	(c)		E.g $m_1 \gg m_2 \Rightarrow \frac{2m_2}{m_1} \approx 0$ or $\frac{49 m_2}{4 m_1} \approx 0$ $\omega \approx \sqrt{\frac{49m}{4m}} = 3.5$	M1 A1 [2]	1.1 1.1	Allow argument such as if $m_1 \gg m_2$ then $m_1 + 2m_2 \approx m_1$ **AG** m may be missing SC1 for result following argument that m_2 is negligible (by comparison with m_1) without justification, or using trial values of m_1 and m_2 with $m_1 \gg m_2$.	Do not allow the assumption that $m_2 = 0$ If using trial values, m_1 must be at least $70 \times m_2$ to give $\omega = 3.5$ to 1dp.

5	(d)		$v = r\omega = 0.6\sqrt{\frac{49\times2.5+98\times2.8}{4\times2.5}}$	M1	1.2	Use of $v = r\omega$ with values for m_1 and m_2	($v = 3.78,\ v^2 = 14.2884$) NB ω = 6.3
			Final energy = $2.5\times g\times1$	B1	1.1	(Assuming zero PE level at 2 m below A; other values possible)	(24.5)
			Initial KE = $\frac{1}{2}\times2.5\times0.6^2\times\frac{49\times2.5+98\times2.8}{4\times2.5}$	M1	1.1	Do not allow use of ω = 3.5	(17.8605)
			Initial PE = $2.5\times g\times1.2+2.8\times g\times0.4$	M1	1.1	oe with different zero PE level	(40.376)
			Energy loss = 17.8605 + 40.376 – 24.5 = 33.7365	A1	3.2a	awrt 33.7	
			Alternate method				
			$v = r\omega = 0.6\sqrt{\frac{49\times2.5+98\times2.8}{4\times2.5}}$	M1		Use of $v = r\omega$ with values for m_1 and m_2	($v = 3.78,\ v^2 = 14.2884$) NB ω = 6.3
			Initial KE = $\frac{1}{2}\times2.5\times0.6^2\times\frac{49\times2.5+98\times2.8}{4\times2.5}$	M1			(17.8605)
			ΔPE for $m_1 = \pm2.5\times9.8\times(0.8-1)$	M1			(±4.9)
			ΔPE for $m_2 = \pm2.8\times9.8(1.6-2)$	M1		Or $-\Delta PE = 2.5\times9.8\times0.2+2.8\times9.8\times0.4$	(±10.976) (±15.876)
			Energy loss = 17.8605 + 4.9 + 10.976	A1		awrt 33.7	Or 15.876 + 17.8605
				[5]			

6	(a)		1st collision for A & B: $2mu = 2mv_A + mv_B$	M1	3.1b	Conservation of momentum	
			$\frac{1}{2} = \frac{v_B - v_A}{u}$	M1	1.1a	Restitution	
			$v_A = \frac{1}{2}u$	A1	1.1		
			2nd collision for A & B: $2m \times \frac{1}{2}u + mU_B = 2mV_A + mV_B$	M1	1.1	Conservation of momentum	May see $-U_B$ or $\pm eu$ Do not allow assumed value of U_B e.g. $\frac{1}{2}u$ or u.
			$\frac{1}{2} = \frac{V_B - V_A}{\frac{1}{2}u - U_B}$	M1	1.1	Restitution	Do not allow assumed value of U_B e.g. $\frac{1}{2}u$ or u.
			$u + U_B = 2V_A + V_B$ and $u - 2U_B = 4V_B - 4V_A$ $\Rightarrow 3u = 6V_B \Rightarrow V_B = \frac{1}{2}u$	A1	2.1	**AG** Intermediate work towards cancellation must be seen	SC1 if assumed value for V_B has been used (giving M0M0), provided $\lvert U_B \rvert \le u$, direction of travel is towards A and equations are otherwise correct.
				[6]			
6	(b)		$v_B = u$	B1	1.1		Award if seen in **(a)**.
			Collision for B & wall: $e = \pm\frac{U_B}{u}$ or $U_B = \pm eu$	M1	3.1b	Restitution May see V_{2B} or similar instead of $\pm eu$ with use of restitution at the end.	Award if seen in **(a)**
			$\frac{\frac{4}{5}d}{\frac{1}{2}u} = \frac{d}{u} + \frac{\frac{1}{5}d}{eu}$	M1	3.1b	Seeing that A travels $\frac{4}{5}d$ at $\frac{1}{2}u$ in the same time as B travels d at u and $\frac{1}{5}d$ at eu	Do not allow assumed rebound speed
			$\frac{3}{5} = \frac{1}{5e}$	M1	1.1	Correctly cancelling d and u and simplifying their 3 term equation including e in the denominator	
			So coefficient of restitution between B and wall is $\frac{1}{3}$	A1	3.2a		
				[5]			

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